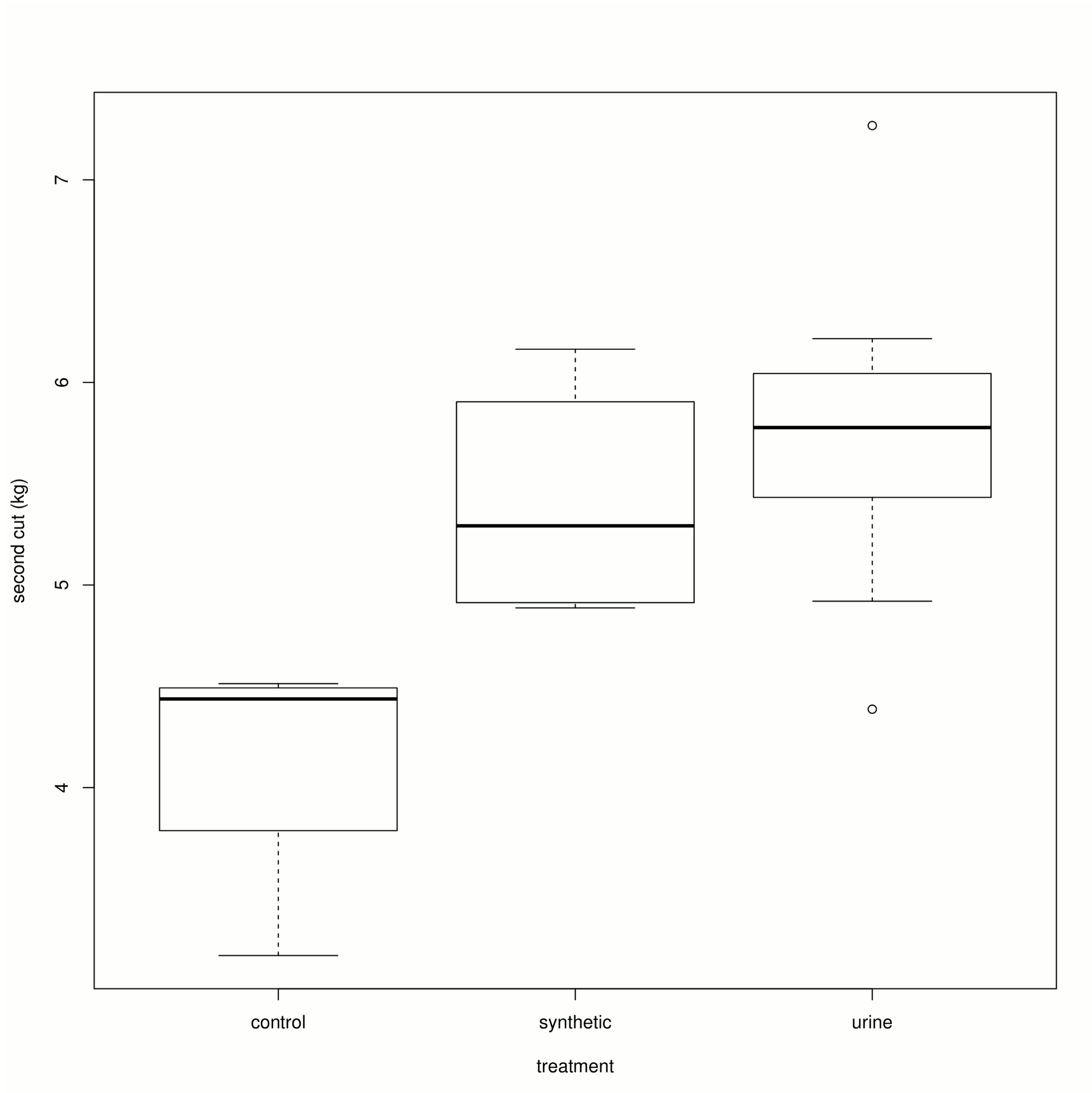
Research questions:

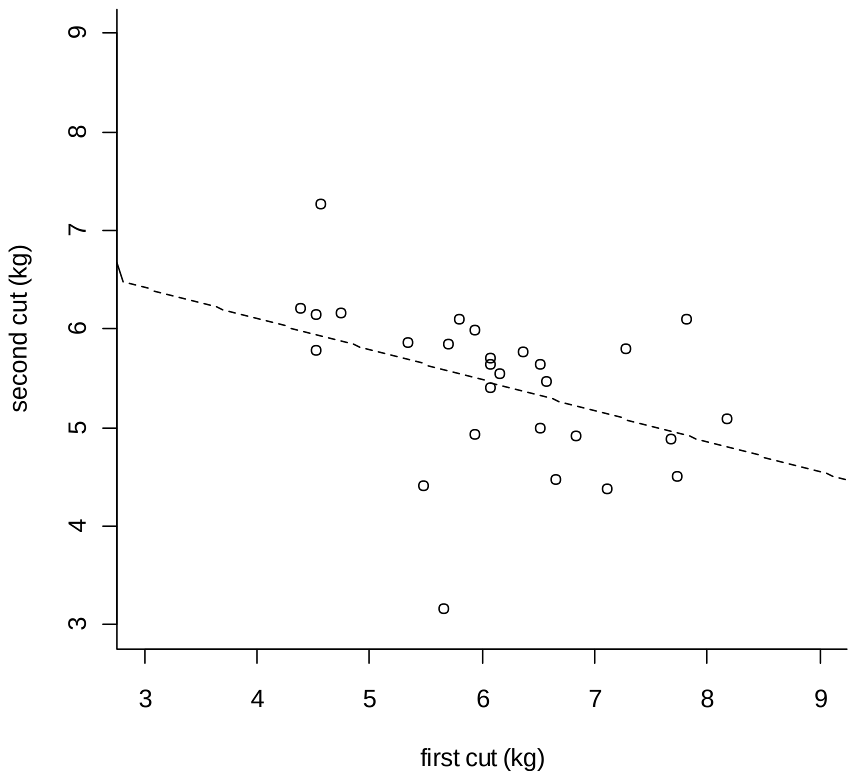
1. Does urine-treatment increase second cutting mass in hay fields in comparison to no fertilizer treatment?
2. Does synthetic fertilizer increase second cutting mass in hay fields in comparison to urine-treated plots?

Exploratory data analysis:

The box plot of second cutting mass versus treatment suggests there is a difference between the control and both the urine-treated and chemical treated plots. It does not suggest a difference between urine-treated and chemical-treated plots. It also suggests that the level of variability within each treatment group is roughly the same, which is a required assumption for the linear models we use below.



We also plot second cutting mass versus first cutting mass below. This scatter, and an empirical correlation of -0.40, suggests there is only a weak relation between these two variables and that the first cutting mass is not a good predictor of the mass obtained in the second cutting. The dashed line shows a simple line of best fit, using first cutting mass as a predictor of second cutting mass. Note that this line goes down.



In terms of numerical summaries, the table below quantifies the graphical impression above: the mass for plots receiving synthetic fertilizer and urine treatments are approximately the same on average, both are higher than the control plot average mass, and the standard deviation of the mass across all treatments is approximately the same.

Treatment	**Control**	**Synthetic fertilizer**	**Urine**
Mean - mass of second cutting (kg)	4.1	5.4	5.7
Standard deviation - mass of second cutting (kg)	0.65	0.61	0.60

Model creation:

To test the preliminary conclusions from the graphs, i.e. to determine whether the observed differences were statistically significant or could plausibly be explained simply by sampling variability in the underlying process, we fit two different models to the data:

The first model uses the mass from the first hay cutting as a proxy for site quality at each plot; it attempts to control for variability in site conditions by fitting a linear term for the first cutting mass and then a fixed effect for the treatment type.

Equation:

$$Y_i = \mu + \beta_{first} X_i + \beta_j + \varepsilon_i$$

Where:

Y_i is the mass of the second cutting in the *i*th plot.

μ is the average mass of the second cutting for untreated plots if the first cutting were equal to the average first cutting mass.

β_{first} is the expected change in the mass of the second cutting for each 1kg change in the mass of the first cutting relative to the average first cutting mass.

X_i is the mass (in kg) of the first cutting in the *i*th plot.

β_j is the expected change in the mass of the second cutting for each treatment (*j* =0 for the control, 1 for the urine-treated plots, and 2 for plots treated with the synthetic fertilizer.).

ε_i is the residual error, i.e. the difference between the observed mass and the predicted mass based on the other terms in the model.

In words, this means the mass of the second cutting is equal to the average mass for untreated plots, plus a site quality adjustment scaled by the mass of the first cutting, plus a treatment effect based on whether the site received no treatment (in which case the treatment effect is zero), urine or synthetic fertilizer, plus an error term that accounts for other sources of variability not measured in the experiment.

Upon fitting this model, statistical tests of the global null hypotheses $\beta_0 = \beta_1 = \beta_2$ as well as the specific pairwise comparisons $\beta_0 = \beta_1$ and $\beta_1 = \beta_2$ will allow us to answer the research questions: Are all of the treatments resulting in equal second cutting mass? Are the second cutting masses from the urine-treated plots different from those in the control plots? And, are the second cutting masses from the urine-treated plots different from those in the plots receiving synthetic chemicals.

The second model does not use first cut mass as a proxy for site quality and instead simply models the main effects for each fertilizer treatment. In equation form, the second model is:

$$Y_i = \alpha + \gamma_j + \varepsilon_i$$

Where:

Y_i is the mass of the second cutting in the *i*th plot.

α is the average mass of the second cutting for the control plots.

γ_j is the expected change in the mass of the second cutting for each treatment (*j* =1 for the urine-treated plots, and 2 for plots treated with the synthetic fertilizer.).

ε_i is the residual error, i.e. the difference between the observed mass and the predicted mass based on the other terms in the model.

There is a slight difference in how this model's parameters are defined: the initial term α in the second model corresponds to the overall mean for the control plots, while the term μ in the first model represented the expected mass for no fertilizer treatment and the average mass across all plots from the first cutting. Consequently, when these models are fit, the estimate for α will be the empirical average for the control plots, while the estimate of μ will not have a similar exact analog from the data, because none of the control plot data points have a first cutting mass exactly equal to the average first cutting mass across all plots.

For the second model, statistical tests of the global null hypotheses $\gamma_1 = \gamma_2 = 0$ as well as the specific pairwise comparisons $\gamma_1 = 0$ and $\gamma_1 = \gamma_2$ will allow us to answer the same research questions outlined regarding the first model above.

The first and second model were both fit with ordinary least squares.

Results

Both models have a numerical term that quantifies the impact of urine treatment on the mass of the second cutting. This numerical value is the change in the average mass of the second cutting from the overall mean across all plots.

Both models show that there is a statistically significant increase in the mass of the second cutting for the urine-treated plots when compared to the control plots. Specifically, the models showed:

Table 1. Model 1 numerical summary

Parameter	Estimated value (standard error)	95% confidence interval
μ (average second cutting kg without treatment and first cutting mass equal to average of all first cuttings kg) – units are kg per plot	4.20 (0.69)	(3.64 , 4.76)
B_1 (change in estimated second cutting mass per kg of first cutting mass) – dimensionless, or can be thought of change in kg of second cutting per kg of difference between first cutting and average of ***all*** first cuttings.	-0.26 (0.10)	(-0.47 , -0.06)
B_{urine} (change in average second cutting kg for plots receiving urine treatment) – units are kg per plot	1.49 (0.30)	(0.87 , 2.10)
$B_{synthetic}$ (change in average second cutting kg for plots receiving synthetic fertilizer treatment) – units are kg per plot	1.23 (0.38)	(0.43 , 2.02)

We note that the effect of the first cutting mass term is negative (-0.26). This suggests an inverse relation between first cutting mass and second cutting mass across all plots. This agrees with visual impression from the earlier scatter plot above showing a slight negative correlation between first cutting mass and second cutting mass.

We also note that the estimates for the urine and synthetic treatments are both significantly different from zero (their 95% confidence intervals do not include zero, indicating they are significant at a α = 0.05 level) and the confidence intervals of these two treatments overlap closely. The numerical estimates suggest that on average, urine treated plots have an increase of about 0.87 to 2.1 kg per plot and synthetic treated plots have an increase of 0.43 to 2.02 kg per plot.

Analysis of the residuals do not show any particular problems with the assumptions used in this model (equal variance of the populations, normality of the populations).

Table 2. Model 2 numerical summary

Parameter	Estimated value (standard error)	95% confidence interval
α (average second cutting kg without treatment – units are kg per plot)	4.1 (0.3)	(3.5 , 4.8)
γ_{urine} (expected change in second cutting kg for urine-treated plots in comparison to control plots – units are kg per plot)	1.56 (0.33)	(0.9,2.2
$\gamma_{synthetic}$ (expected change in second cutting kg for plots with synthetic fertilizer in comparison to control plots – units are kg per plot)	1.27 (0.43)	(0.4,2.2)

In both models, hypothesis testing shows a statistically significant difference between urine-treated and control plots, and does not show a statistically significant difference between urine-treated and plots treated with synthetic fertilizer. The magnitude of this difference is an increase of approximately 1.5 kg in comparison to a control average second cutting mass of 4.1 kg, hence an increase of approximately 37% +/- 8%, representing one standard error.